

# Package ‘spBPS’

March 19, 2026

**Title** Bayesian Predictive Stacking for Scalable Geospatial Transfer Learning

**Version** 1.0-1

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**Description** Provides functions for Bayesian Predictive Stacking within the Bayesian transfer learning framework for geospatial artificial systems, as introduced in “Bayesian Transfer Learning for Artificially Intelligent Geospatial Systems: A Predictive Stacking Approach” (Presicce and Banerjee, 2025) <[doi:10.48550/arXiv.2410.09504](https://doi.org/10.48550/arXiv.2410.09504)>. This methodology enables efficient Bayesian geostatistical modeling, utilizing predictive stacking to improve inference across spatial datasets. The core functions leverage ‘C++’ for high-performance computation, making the framework well-suited for large-scale spatial data analysis in parallel and distributed computing environments. Designed for scalability, it allows seamless application in computationally demanding scenarios.

**Depends** R (>= 1.8.0)

**Imports** Rcpp, CVXR (>= 1.8.1), mniw

**LinkingTo** Rcpp, RcppArmadillo

**Suggests** knitr, rmarkdown, abind, mvnfast, ECOSolveR, foreach,  
parallel, doParallel, tictoc, MBA, RColorBrewer, classInt, sp,  
fields, testthat (>= 3.0.0)

**Config/testthat/edition** 3

**License** GPL (>= 3)

**Encoding** UTF-8

**RoxygenNote** 7.3.3

**VignetteBuilder** knitr

**URL** <https://lucapresicce.github.io/spBPS/>

**NeedsCompilation** yes

**Repository** CRAN

**Date/Publication** 2026-03-19 15:30:02 UTC

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arma_dist	<i>Compute the Euclidean distance matrix</i>
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### Description

Compute the Euclidean distance matrix

### Usage

arma\_dist(X)

### Arguments

X [matrix](#) (typically of  $N$  coordinates on  $\mathbb{R}^2$ )

### Value

[matrix](#) distance matrix of the elements of  $X$

---

bayesMvLMconjugate      *Gibbs sampler for Conjugate Bayesian Multivariate Linear Models*

---

## Description

Gibbs sampler for Conjugate Bayesian Multivariate Linear Models

## Usage

```
bayesMvLMconjugate(Y, X, mu_B, V_B, nu, Psi, n_iter = 1000, burn_in = 500)
```

## Arguments

Y	matrix $n \times q$ of response variables
X	matrix $n \times p$ of predictors
mu_B	matrix $p \times q$ prior mean for $\beta$
V_B	matrix $p \times p$ prior row covariance for $\beta$
nu	double prior parameter for $\Sigma$
Psi	matrix prior parameter for $\Sigma$
n_iter	integer iteration number for Gibbs sampler
burn_in	integer number of burn-in iteration

## Value

B\_samples array of posterior sample for  $\beta$   
Sigma\_samples array of posterior samples for  $\Sigma$

## Examples

```
## Generate data
n <- 100
p <- 3
q <- 2
Y <- matrix(rnorm(n*q), nrow = n, ncol = q)
X <- matrix(rnorm(n*p), nrow = n, ncol = p)

## Prior parameters
mu_B <- matrix(0, p, q)
V_B <- diag(10, p)
nu <- 3
Psi <- diag(q)

## Samples from posteriors
n_iter <- 1000
burn_in <- 500
set.seed(1234)
```

```
samples <- spBPS::bayesMvLMconjugate(Y, X, mu_B, V_B, nu, Psi, n_iter, burn_in)
```

---

BPS\_combine                      *Combine subset models wiht BPS*

---

### Description

Combine subset models wiht BPS

### Usage

```
BPS_combine(fit_list, K, rp)
```

### Arguments

fit_list	<b>list</b> K fitted model outputs composed by two elements each: first named <i>epd</i> , second named <i>W</i>
K	<b>integer</b> number of folds
rp	<b>double</b> percentage of observations to take into account for optimization (default=1)

### Value

**matrix** posterior predictive density evaluations (each columns represent a different model)

---

BPS\_postdraws\_MvT                      *Compute the BPS posterior samples given a set of stacking weights*

---

### Description

Compute the BPS posterior samples given a set of stacking weights

### Usage

```
BPS_postdraws_MvT(data, priors, coords, hyperpar, W, R, par)
```

### Arguments

data	<b>list</b> two elements: first named <i>Y</i> , second named <i>X</i>
priors	<b>list</b> priors: named $\mu_B, V_r, \Psi, \nu$
coords	<b>matrix</b> sample coordinates for X and Y
hyperpar	<b>list</b> two elemets: first named $\alpha$ , second named $\phi$
W	<b>matrix</b> set of stacking weights
R	<b>integer</b> number of desired samples
par	if TRUE only $\beta$ and $\Sigma$ are sampled ( $\omega$ is omitted)

**Value**

**matrix** BPS posterior samples

---

BPS_post_MvT	<i>Perform the BPS sampling from posterior and posterior predictive given a set of stacking weights</i>
--------------	---

---

**Description**

Perform the BPS sampling from posterior and posterior predictive given a set of stacking weights

**Usage**

BPS\_post\_MvT(data, X\_u, priors, coords, crd\_u, hyperpar, W, R)

**Arguments**

data	<b>list</b> two elements: first named $Y$ , second named $X$
X_u	<b>matrix</b> unobserved instances covariate matrix
priors	<b>list</b> priors: named $\mu_B, V_r, \Psi, \nu$
coords	<b>matrix</b> sample coordinates for X and Y
crd_u	<b>matrix</b> unobserved instances coordinates
hyperpar	<b>list</b> two elements: first named $\alpha$ , second named $\phi$
W	<b>matrix</b> set of stacking weights
R	<b>integer</b> number of desired samples

**Value**

**list** BPS posterior predictive samples

---

BPS_pred_MvT	<i>Compute the BPS spatial prediction given a set of stacking weights</i>
--------------	---

---

**Description**

Compute the BPS spatial prediction given a set of stacking weights

**Usage**

BPS\_pred\_MvT(data, X\_u, priors, coords, crd\_u, hyperpar, W, R)

**Arguments**

data	<b>list</b> two elements: first named $Y$ , second named $X$
X_u	<b>matrix</b> unobserved instances covariate matrix
priors	<b>list</b> priors: named $\mu_B, V_r, \Psi, \nu$
coords	<b>matrix</b> sample coordinates for $X$ and $Y$
crd_u	<b>matrix</b> unobserved instances coordinates
hyperpar	<b>list</b> two elements: first named $\alpha$ , second named $\phi$
W	<b>matrix</b> set of stacking weights
R	<b>integer</b> number of desired samples

**Value**

**list** BPS posterior predictive samples

---

BPS\_PseudoBMA

*Combine subset models with Pseudo-BMA*

---

**Description**

Combine subset models with Pseudo-BMA

**Usage**

BPS\_PseudoBMA(fit\_list)

**Arguments**

fit_list	<b>list</b> $K$ fitted model outputs composed by two elements each: first named $epd$ , second named $W$
----------	--

**Value**

**matrix** posterior predictive density evaluations (each column represents a different model)

---

BPS_weights_MvT	<i>Compute the BPS weights by convex optimization</i>
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**Description**

Compute the BPS weights by convex optimization

**Usage**

BPS\_weights\_MvT(data, priors, coords, hyperpar, K)

**Arguments**

data	<b>list</b> two elements: first named $Y$ , second named $X$
priors	<b>list</b> priors: named $\mu_B, V_r, \Psi, \nu$
coords	<b>matrix</b> sample coordinates for $X$ and $Y$
hyperpar	<b>list</b> two elements: first named $\alpha$ , second named $\phi$
K	<b>integer</b> number of folds

**Value**

**matrix** posterior predictive density evaluations (each column represent a different model)

---

conv_opt	<i>Solver for Bayesian Predictive Stacking of Predictive densities convex optimization problem</i>
----------	--

---

**Description**

Solver for Bayesian Predictive Stacking of Predictive densities convex optimization problem

**Usage**

conv\_opt(scores)

**Arguments**

scores	<b>matrix</b> $N \times K$ of expected predictive density evaluations for the $K$ models considered
--------	---

**Value**

$W$  **matrix** of Bayesian Predictive Stacking weights for the  $K$  models considered

---

CVXR_opt	<i>Compute the BPS weights by convex optimization</i>
----------	---

---

**Description**

Compute the BPS weights by convex optimization

**Usage**

CVXR\_opt(scores)

**Arguments**

scores      **matrix**  $N \times K$  of expected predictive density evaluations for the K models considered

**Value**

conv\_opt **function** to perform convex optimization with CVXR R package

---

dens_kcv_MvT	<i>Compute the KCV of the density evaluations for fixed values of the hyperparameters</i>
--------------	---

---

**Description**

Compute the KCV of the density evaluations for fixed values of the hyperparameters

**Usage**

dens\_kcv\_MvT(data, priors, coords, hyperpar, K)

**Arguments**

data      **list** two elements: first named  $Y$ , second named  $X$   
 priors    **list** priors: named  $\mu_B, V_r, \Psi, \nu$   
 coords    **matrix** sample coordinates for  $X$  and  $Y$   
 hyperpar **list** two elements: first named  $\alpha$ , second named  $\phi$   
 K         **integer** number of folds

**Value**

**vector** posterior predictive density evaluations

---

dens_loocv_MvT	<i>Compute the LOOCV of the density evaluations for fixed values of the hyperparameters</i>
----------------	---

---

**Description**

Compute the LOOCV of the density evaluations for fixed values of the hyperparameters

**Usage**

```
dens_loocv_MvT(data, priors, coords, hyperpar)
```

**Arguments**

data	<b>list</b> two elements: first named $Y$ , second named $X$
priors	<b>list</b> priors: named $\mu_B, V_r, \Psi, \nu$
coords	<b>matrix</b> sample coordinates for $X$ and $Y$
hyperpar	<b>list</b> two elements: first named $\alpha$ , second named $\phi$

**Value**

**vector** posterior predictive density evaluations

---

d_pred_cpp_MvT	<i>Evaluate the density of a set of unobserved response with respect to the conditional posterior predictive</i>
----------------	--

---

**Description**

Evaluate the density of a set of unobserved response with respect to the conditional posterior predictive

**Usage**

```
d_pred_cpp_MvT(data, X_u, Y_u, d_u, d_us, hyperpar, poster)
```

**Arguments**

data	<b>list</b> two elements: first named $Y$ , second named $X$
$X_u$	<b>matrix</b> unobserved instances covariate matrix
$Y_u$	<b>matrix</b> unobserved instances response matrix
$d_u$	<b>matrix</b> unobserved instances distance matrix
$d_{us}$	<b>matrix</b> cross-distance between unobserved and observed instances matrix
hyperpar	<b>list</b> two elements: first named $\alpha$ , second named $\phi$
poster	<b>list</b> output from <code>fit_cpp</code> function

**Value**

**double** posterior predictive density evaluation

---

expand_grid_cpp	<i>Build a grid from two vector (i.e. equivalent to expand.grid() in R)</i>
-----------------	---

---

**Description**

Build a grid from two vector (i.e. equivalent to `expand.grid()` in R)

**Usage**

```
expand_grid_cpp(x, y)
```

**Arguments**

x	<b>vector</b> first vector of numeric elements
y	<b>vector</b> second vector of numeric elements

**Value**

**matrix** expanded grid of combinations

---

fit_cpp_MvT	<i>Compute the parameters for the posteriors distribution of <math>\beta</math> and <math>\Sigma</math> (i.e. updated parameters)</i>
-------------	---

---

**Description**

Compute the parameters for the posteriors distribution of  $\beta$  and  $\Sigma$  (i.e. updated parameters)

**Usage**

```
fit_cpp_MvT(data, priors, coords, hyperpar)
```

**Arguments**

data	<b>list</b> two elements: first named $Y$ , second named $X$
priors	<b>list</b> priors: named $\mu_B, V_r, \Psi, \nu$
coords	<b>matrix</b> sample coordinates for $X$ and $Y$
hyperpar	<b>list</b> two elemets: first named $\alpha$ , second named $\phi$

**Value**

**list** posterior update parameters

---

forceSymmetry_cpp	<i>Function to subset data for meta-analysis</i>
-------------------	--

---

**Description**

Function to subset data for meta-analysis

**Usage**

```
forceSymmetry_cpp(mat)
```

**Arguments**

mat                    **matrix** not-symmetric matrix

**Value**

**matrix** symmetric matrix (lower triangular of mat is used)

---

models_dens_MvT	<i>Return the CV predictive density evaluations for all the model combinations</i>
-----------------	--

---

**Description**

Return the CV predictive density evaluations for all the model combinations

**Usage**

```
models_dens_MvT(data, priors, coords, hyperpar, useKCV, K)
```

**Arguments**

data                    **list** two elements: first named  $Y$ , second named  $X$   
priors                    **list** priors: named  $\mu_B, V_r, \Psi, \nu$   
coords                    **matrix** sample coordinates for  $X$  and  $Y$   
hyperpar                    **list** two elemets: first named  $\alpha$ , second named  $\phi$   
useKCV                    if TRUE K-fold cross validation is used instead of LOOCV (no default)  
K                         **integer** number of folds

**Value**

**matrix** posterior predictive density evaluations (each columns represent a different model)



**Examples**

```

## Generate data
n <- 100
p <- 3
q <- 2
Y <- matrix(rnorm(n*q), nrow = n, ncol = q)
X <- matrix(rnorm(n*p), nrow = n, ncol = p)

## Prior parameters
mu_B <- matrix(0, p, q)
V_B <- diag(10, p)
nu <- 3
Psi <- diag(q)

## Samples from posteriors
n_iter <- 1000
burn_in <- 500
set.seed(1234)
samples <- spBPS::bayesMvLMconjugate(Y, X, mu_B, V_B, nu, Psi, n_iter, burn_in)

## Extract posterior samples
B_samples <- samples$B_samples
Sigma_samples <- samples$Sigma_samples

## Samples from predictive posterior (based posterior samples)
m <- 50
X_new <- matrix(rnorm(m*p), nrow = m, ncol = p)
pred <- spBPS::pred_bayesMvLMconjugate(X_new, B_samples, Sigma_samples)

```

---

r_pred_cond_MvT	<i>Draw from the conditional posterior predictive for a set of unobserved covariates</i>
-----------------	--

---

**Description**

Draw from the conditional posterior predictive for a set of unobserved covariates

**Usage**

```
r_pred_cond_MvT(data, X_u, d_u, d_us, hyperpar, poster, post)
```

**Arguments**

data	<b>list</b> two elements: first named $Y$ , second named $X$
X_u	<b>matrix</b> unobserved instances covariate matrix
d_u	<b>matrix</b> unobserved instances distance matrix

d_us	<b>matrix</b> cross-distance between unobserved and observed instances matrix
hyperpar	<b>list</b> two elemets: first named $\alpha$ , second named $\phi$
poster	<b>list</b> output from fit_cpp_MvT function
post	<b>list</b> output from post_draws_MvT function

**Value**

**list** posterior predictive samples

---

r_pred_joint_MvT	<i>Draw from the joint posterior predictive for a set of unobserved covariates</i>
------------------	--

---

**Description**

Draw from the joint posterior predictive for a set of unobserved covariates

**Usage**

```
r_pred_joint_MvT(data, X_u, d_u, d_us, hyperpar, poster, R)
```

**Arguments**

data	<b>list</b> two elements: first named $Y$ , second named $X$
X_u	<b>matrix</b> unobserved instances covariate matrix
d_u	<b>matrix</b> unobserved instances distance matrix
d_us	<b>matrix</b> cross-distance between unobserved and observed instances matrix
hyperpar	<b>list</b> two elemets: first named $\alpha$ , second named $\phi$
poster	<b>list</b> output from fit_cpp function
R	<b>integer</b> number of posterior predictive samples

**Value**

**list** posterior predictive samples

---

r_pred_marg_MvT	<i>Draw from the joint posterior predictive for a set of unobserved covariates</i>
-----------------	--

---

**Description**

Draw from the joint posterior predictive for a set of unobserved covariates

**Usage**

```
r_pred_marg_MvT(data, X_u, d_u, d_us, hyperpar, poster, R)
```

**Arguments**

data	<b>list</b> two elements: first named $Y$ , second named $X$
X_u	<b>matrix</b> unobserved instances covariate matrix
d_u	<b>matrix</b> unobserved instances distance matrix
d_us	<b>matrix</b> cross-distance between unobserved and observed instances matrix
hyperpar	<b>list</b> two elements: first named $\alpha$ , second named $\phi$
poster	<b>list</b> output from <code>fit_cpp</code> function
R	<b>integer</b> number of posterior predictive samples

**Value**

**list** posterior predictive samples

---

sample_index	<i>Function to sample integers (index)</i>
--------------	--

---

**Description**

Function to sample integers (index)

**Usage**

```
sample_index(size, length, p)
```

**Arguments**

size	<b>integer</b> dimension of the set to sample
length	<b>integer</b> number of elements to sample
p	<b>vector</b> sampling probabilities

**Value**

**vector** sample of integers

spBPS

*Unified spatial BPS workflow (multivariate path, works for  $q = 1$ )***Description**

Orchestrates subsetting, local stacking weight estimation, global stacking combination, and optional posterior or predictive simulation using the multivariate Student-t spatial model. Works for both multivariate outcomes and the univariate case via  $q = 1$ .

**Usage**

```
spBPS(
  data,
  priors,
  coords,
  hyperpar,
  subset_size = 500L,
  K = NULL,
  cv_folds = 5L,
  rp = 1,
  combine_method = c("bps", "pseudoBMA"),
  draws = 0L,
  newdata = NULL,
  include_latent = FALSE,
  cores = NULL
)
```

**Arguments**

<code>data</code>	List with matrices $Y$ (response) and $X$ (covariates).
<code>priors</code>	List of priors for the multivariate model ( $\mu_B, V_r, \Psi, \nu$ ).
<code>coords</code>	Matrix of observation coordinates.
<code>hyperpar</code>	List with elements $\alpha$ and $\phi$ (vectors allowed).
<code>subset_size</code>	Target subset size when $K$ is not provided. Default 500.
<code>K</code>	Optional number of subsets. When <code>NULL</code> , computed as $\text{ceiling}(\text{nrow}(Y) / \text{subset\_size})$ and lower-bounded at 1.
<code>cv_folds</code>	Number of folds for local cross-validation (default 5).
<code>rp</code>	Fraction of rows used when recomputing global stacking weights (passed to <code>BPS_combine</code> ). Ignored when <code>combine_method = "pseudoBMA"</code> .
<code>combine_method</code>	Choose between Bayesian Predictive Stacking (" <code>bps</code> ") or pseudo-BMA (" <code>pseudoBMA</code> ") for combining subsets.
<code>draws</code>	Number of joint posterior/predictive draws to return (0 to skip). When positive, <code>newdata</code> must be supplied because draws are obtained via <code>BPS_post_MvT</code> which jointly samples posterior and predictive.

newdata	Optional list with $X$ and coords for prediction locations; required when either draw count is positive.
include_latent	Logical; if TRUE, posterior draws include latent processes.
cores	Optional integer; when $>1$ a parallel backend is registered internally via <code>doParallel::registerDoParallel</code> for the fit and draw loops. When NULL, the existing foreach backend (if any) is used.

**Value**

List with components `subsets`, `weights_global`, `weights_local`, `epd`, and optional posterior and predictive draws.

**Examples**

```
n <- 1000
p <- 2
q <- 1

Y <- matrix(rnorm(n*q), ncol = q)
X <- matrix(rnorm(n*p), ncol = p)
coords <- matrix(runif(n*2), ncol = 2)

data <- list(Y = Y, X = X)
priors <- list(mu_B = matrix(0, nrow = p, ncol = q),
              V_r = diag(10, p),
              Psi = diag(1, q),
              nu = 3)
hyperpar <- list(alpha = 0.5, phi = 1)
subset_size <- 200

res <- spBPS(data, priors, coords, hyperpar, subset_size = subset_size)
```

---

subset\_data

*Function to subset data for meta-analysis*


---

**Description**

Function to subset data for meta-analysis

**Usage**

```
subset_data(data, K)
```

**Arguments**

`data` [list](#) three elements: first named  $Y$ , second named  $X$ , third named *crd*  
`K` [integer](#) number of desired subsets

**Value**

[list](#) subsets of data, and the set of indexes

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