

Package ‘MultiBD’

March 24, 2026

Type Package

Title Multivariate Birth-Death Processes

Version 1.0.2

Date 2026-03-23

Maintainer Marc A. Suchard <msuchard@ucla.edu>

Description Computationally efficient functions to provide direct likelihood-based inference for partially-observed multivariate birth-death processes. Such processes range from a simple Yule model to the complex susceptible-infectious-removed model in disease dynamics. Efficient likelihood evaluation facilitates maximum likelihood estimation and Bayesian inference.

License Apache License 2.0

Depends R (>= 3.1.0)

Imports Rcpp (>= 0.11.2), RcppParallel

LinkingTo Rcpp, BH, RcppParallel

Suggests testthat, knitr, rmarkdown, MCMCpack, ggplot2, matrixStats, plotrix

RoxygenNote 7.3.2

VignetteBuilder knitr

Encoding UTF-8

NeedsCompilation yes

Author Lam S.T. Ho [aut],
Marc A. Suchard [aut, cre],
Forrest W. Crawford [aut],
Jason Xu [ctb],
Vladimir N. Minin [ctb]

Repository CRAN

Date/Publication 2026-03-24 16:40:03 UTC

Contents

bbd_prob	2
dbd_prob	4
Eyam	6
MultiBD	7
SEIR_prob	7
SIR_prob	8

Index	11
--------------	-----------

bbd_prob	<i>Transition probabilities of a birth/birth-death process</i>
----------	--

Description

Computes the transition probabilities of a birth/birth-death process using the continued fraction representation of its Laplace transform

Usage

```
bbd_prob(
  t,
  a0,
  b0,
  lambda1,
  lambda2,
  mu2,
  gamma,
  A,
  B,
  nblocks = 256,
  tol = 1e-12,
  computeMode = 0,
  nThreads = 4,
  maxdepth = 400
)
```

Arguments

t	time
a0	total number of type 1 particles at $t = 0$
b0	total number of type 2 particles at $t = 0$
lambda1	birth rate of type 1 particles (a two variables function)
lambda2	birth rate of type 2 particles (a two variables function)
mu2	death rate function of type 2 particles (a two variables function)

gamma	transition rate from type 2 particles to type 1 particles (a two variables function)
A	upper bound for the total number of type 1 particles
B	upper bound for the total number of type 2 particles
nblocks	number of blocks
tol	tolerance
computeMode	computation mode
nThreads	number of threads
maxdepth	maximum number of iterations for Lentz algorithm

Value

a matrix of the transition probabilities

References

Ho LST et al. 2015. "Birth(death)/birth-death processes and their computable transition probabilities with statistical applications". In review.

Examples

```
## Not run:
data(Eyam)

# (R, I) in the SIR model forms a birth/birth-death process

loglik_sir <- function(param, data) {
  alpha <- exp(param[1]) # Rates must be non-negative
  beta <- exp(param[2])
  N <- data$S[1] + data$I[1] + data$R[1]

  # Set-up SIR model with (R, I)

  brates1 <- function(a, b) { 0 }
  brates2 <- function(a, b) { beta * max(N - a - b, 0) * b }
  drates2 <- function(a, b) { 0 }
  trans21 <- function(a, b) { alpha * b }

  sum(sapply(1:(nrow(data) - 1), # Sum across all time steps k
           function(k) {
             log(
               bbd_prob( # Compute the transition probability matrix
                 t = data$time[k + 1] - data$time[k], # Time increment
                 a0 = data$R[k], b0 = data$I[k], # From: R(t_k), I(t_k)
                 brates1, brates2, drates2, trans21,
                 A = data$R[k + 1], B = data$R[k + 1] + data$I[k] - data$R[k],
                 computeMode = 4, nblocks = 80 # Compute using 4 threads
               )[data$R[k + 1] - data$R[k] + 1,
                data$I[k + 1] + 1] # To: R(t_(k+1)), I(t_(k+1))
             )
           })
  )))
```

```

}

loglik_sir(log(c(3.204, 0.019)), Eyam) # Evaluate at mode

## End(Not run)

```

dbd_prob

Transition probabilities of a death/birth-death process

Description

Computes the transition probabilities of a death/birth-death process using the continued fraction representation of its Laplace transform

Usage

```

dbd_prob(
  t,
  a0,
  b0,
  mu1,
  lambda2,
  mu2,
  gamma,
  a = 0,
  B,
  nblocks = 256,
  tol = 1e-12,
  computeMode = 0,
  nThreads = 4,
  maxdepth = 400
)

```

Arguments

t	time
a0	total number of type 1 particles at $t = 0$
b0	total number of type 2 particles at $t = 0$
mu1	death rate of type 1 particles (a two variables function)
lambda2	birth rate of type 2 particles (a two variables function)
mu2	death rate function of type 2 particles (a two variables function)
gamma	transition rate from type 2 particles to type 1 particles (a two variables function)
a	lower bound for the total number of type 1 particles (default $a = 0$)
B	upper bound for the total number of type 2 particles
nblocks	number of blocks

tol	tolerance
computeMode	computation mode
nThreads	number of threads
maxdepth	maximum number of iterations for Lentz algorithm

Value

a matrix of the transition probabilities

References

Ho LST et al. 2016. "Birth(death)/birth-death processes and their computable transition probabilities with statistical applications". In review.

Examples

```
## Not run:
data(Eyam)

loglik_sir <- function(param, data) {
  alpha <- exp(param[1]) # Rates must be non-negative
  beta  <- exp(param[2])

  # Set-up SIR model
  drates1 <- function(a, b) { 0 }
  brates2 <- function(a, b) { 0 }
  drates2 <- function(a, b) { alpha * b }
  trans12 <- function(a, b) { beta * a * b }

  sum(sapply(1:(nrow(data) - 1), # Sum across all time steps k
           function(k) {
             log(
               dbd_prob( # Compute the transition probability matrix
                 t = data$time[k + 1] - data$time[k], # Time increment
                 a0 = data$S[k], b0 = data$I[k],      # From: S(t_k), I(t_k)
                 drates1, brates2, drates2, trans12,
                 a = data$S[k + 1], B = data$S[k] + data$I[k] - data$S[k + 1],
                 computeMode = 4, nblocks = 80       # Compute using 4 threads
               )[1, data$I[k + 1] + 1]                # To: S(t_(k+1)), I(t_(k+1))
             )
           })
  )
}

loglik_sir(log(c(3.204, 0.019)), Eyam) # Evaluate at mode

## End(Not run)

# Birth-death-shift model for transposable elements

lam = 0.0188; mu = 0.0147; v = 0.00268; # birth, death, shift rates
```

```
drates1 <- function(a, b) { mu * a }
brates2 <- function(a, b) { lam * (a + b) }
drates2 <- function(a, b) { mu * b }
trans12 <- function(a, b) { v * a }

# Get transition probabilities
p <- dbd_prob(t = 1, a0 = 10, b0 = 0,
              drates1, brates2, drates2, trans12,
              a = 0, B = 50)
```

Eyam

Eyam plague.

Description

A dataset containing the number of susceptible, infectious and removed individuals during the Eyam plague from June 18 to October 20, 1666.

Usage

```
data(Eyam)
```

Format

A data frame with 8 rows and 4 variables:

time Months past June 18 1666

S Susceptible

I Infectious

R Removed

References

Ragget G (1982). A stochastic model of the Eyam plague. *Journal of Applied Statistics* 9, 212-226.

MultiBD

Multivariate birth-death processes

Description

The MultiBD package computes the transition probabilities of several multivariate birth-death processes.

Author(s)

Maintainer: Marc A. Suchard <msuchard@ucla.edu>

Authors:

- Lam S.T. Ho
- Forrest W. Crawford

Other contributors:

- Jason Xu [contributor]
- Vladimir N. Minin [contributor]

References

Ho LST et al. 2016. "Birth(death)/birth-death processes and their computable transition probabilities with statistical applications". In review.

SEIR_prob

Transition probabilities of an SEIR process

Description

Computes the transition probabilities of an SIR process using the bivariate birth process representation

Usage

```
SEIR_prob(  
  t,  
  alpha,  
  beta,  
  kappa,  
  S0,  
  E0,  
  I0,  
  nSE,
```

```

nEI,
nIR,
direction = c("Forward", "Backward"),
nblocks = 20,
tol = 1e-12,
computeMode = 0,
nThreads = 4
)

```

Arguments

t	time
alpha	removal rate
beta	infection rate
kappa	rate at which an exposed person becomes infective
S0	initial susceptible population
E0	initial exposed population
I0	initial infectious population
nSE	number of infection events
nEI	number of events at which an exposed person becomes infective
nIR	number of removal events
direction	direction of the transition probabilities (either Forward or Backward)
nblocks	number of blocks
tol	tolerance
computeMode	computation mode
nThreads	number of threads

Value

a matrix of the transition probabilities

SIR_prob

Transition probabilities of an SIR process

Description

Computes the transition probabilities of an SIR process using the bivariate birth process representation

Usage

```

SIR_prob(
  t,
  alpha,
  beta,
  S0,
  I0,
  nSI,
  nIR,
  direction = c("Forward", "Backward"),
  power = NULL,
  nblocks = 20,
  tol = 1e-12,
  computeMode = 0,
  nThreads = 4
)

```

Arguments

t	time
alpha	removal rate
beta	infection rate
S0	initial susceptible population
I0	initial infectious population
nSI	number of infection events
nIR	number of removal events
direction	direction of the transition probabilities (either Forward or Backward)
power	the power of the general SIR model (see Note for more details)
nblocks	number of blocks
tol	tolerance
computeMode	computation mode
nThreads	number of threads

Value

a matrix of the transition probabilities

Note

The infection rate and the removal rate of a general SIR model are $\beta * S^{\text{powS}} * I^{\text{powI_inf}}$ and $\alpha * I^{\text{powI_rem}}$ respectively. The parameter power is a list of powS, powI_inf, powI_rem. Their default values are powS = powI_inf = powI_rem = 1, which correspond to the classic SIR model.

Examples

```

data(Eyam)

loglik_sir <- function(param, data) {
  alpha <- exp(param[1]) # Rates must be non-negative
  beta  <- exp(param[2])

  if(length(unique(rowSums(data[, c("S", "I", "R")])))) > 1) {
    stop ("Please make sure the data conform with a closed population")
  }

  sum(sapply(1:(nrow(data) - 1), # Sum across all time steps k
           function(k) {
             log(
               SIR_prob( # Compute the forward transition probability matrix
                 t = data$time[k + 1] - data$time[k], # Time increment
                 alpha = alpha, beta = beta,
                 S0 = data$S[k], I0 = data$I[k],          # From: R(t_k), I(t_k)
                 nSI = data$S[k] - data$S[k + 1], nIR = data$R[k + 1] - data$R[k],
                 computeMode = 4, nblocks = 80           # Compute using 4 threads
               )[data$S[k] - data$S[k + 1] + 1,
                data$R[k + 1] - data$R[k] + 1]          # To: R(t_(k+1)), I(t_(k+1))
             )
           })
  )
}

loglik_sir(log(c(3.204, 0.019)), Eyam) # Evaluate at mode

```

Index

bbd_prob, [2](#)

dbd_prob, [4](#)

Eyam, [6](#)

MultiBD, [7](#)

MultiBD-package (MultiBD), [7](#)

SEIR_prob, [7](#)

SIR_prob, [8](#)